Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 30 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1.

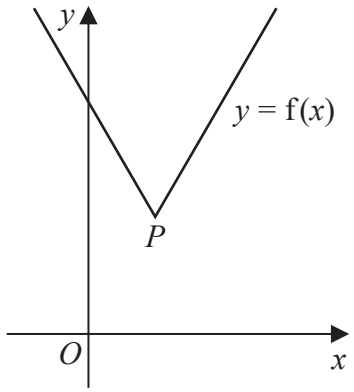

Figure 1

Figure 1 shows a sketch of the graph with equation $y = f(x)$ where

$$f(x) = 2|x - 5| + 10$$

The point P, shown in Figure 1, is the vertex of the graph.

(a) State the coordinates of P

(2)

(b) Use algebra to solve

$$2|x - 5| + 10 > 6x$$

(*Solutions relying on calculator technology are not acceptable.*)

(2)

(c) Find the point to which P is mapped, when the graph with equation $y = f(x)$ is transformed to the graph with equation $y = 3f(x - 2)$

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2. $$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$$

(a) Write $g(x)$ in the form

$$Ax + B + \frac{C}{x - 2}$$

where A, B and C are integers to be found.

(3)

(b) Hence use algebraic integration to show that

$$\int_4^8 g(x)\,dx = \alpha + \beta \ln 3$$

where α and β are integers to be found.

(4)

Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 7 marks)

3. (i) The variables x and y are connected by the equation

$$y = \frac{10^6}{x^3} \qquad x > 0$$

Sketch the graph of $\log_{10} y$ against $\log_{10} x$

Show on your sketch the coordinates of the points of intersection of the graph with the axes.

(3)

(ii)

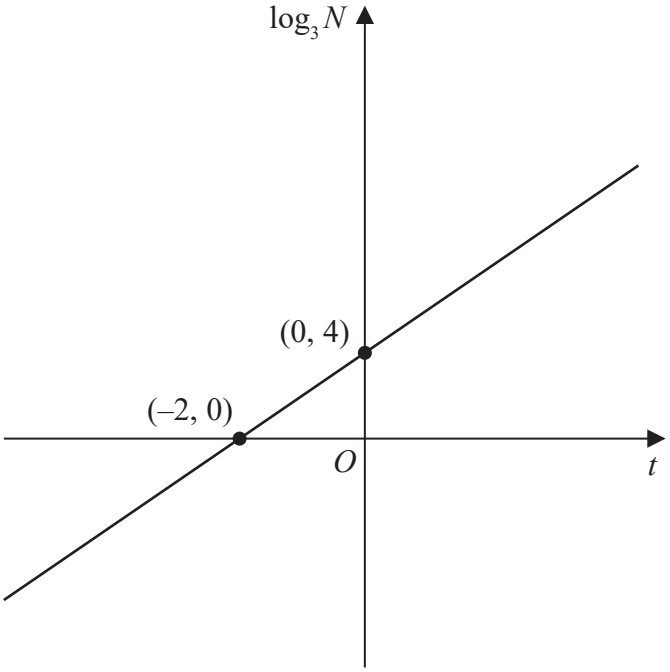

Figure 2

Figure 2 shows the linear relationship between $\log_3 N$ and t.

Show that $N = ab^t$ where a and b are constants to be found.

(3)

Question 3 continued

(Total for Question 3 is 6 marks)

4. $f(x) = 8\sin x\cos x + 4\cos^2 x - 3$

(a) Write $f(x)$ in the form

$$a\sin 2x + b\cos 2x + c$$

where a, b and c are integers to be found.

(3)

(b) Use the answer to part (a) to write $f(x)$ in the form

$$R\sin(2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 significant figures.

(3)

(c) Hence, or otherwise,

(i) state the maximum value of $f(x)$

(ii) find the **second** smallest positive value of x at which a maximum value of $f(x)$ occurs. Give your answer to 3 significant figures.

(3)

Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

5. The functions f and g are defined by

$$f(x) = 2 + 5\ln x \qquad x > 0$$

$$g(x) = \frac{6x - 2}{2x + 1} \qquad x > \frac{1}{3}$$

(a) Find $f^{-1}(22)$

(2)

(b) Use differentiation to prove that g is an increasing function.

(3)

(c) Find g^{-1}

(3)

(d) Find the range of fg

(2)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

6.

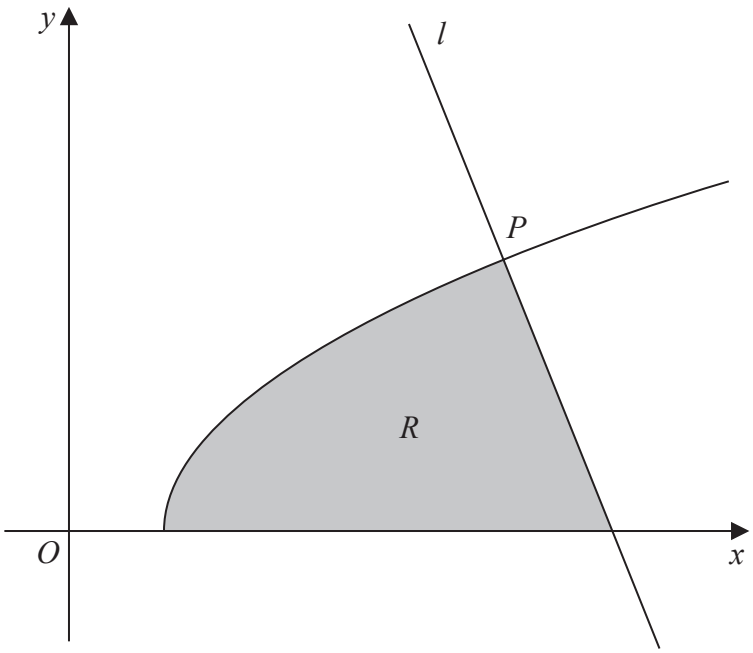

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and l.

(b) Use algebraic integration to find the exact area of R.

(4)

Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$\sqrt{2}\sin(x + 45°) = \cos(x - 60°)$$

show that

$$\tan x = -2 - \sqrt{3}$$

(4)

(b) Hence or otherwise, solve, for $0 \leqslant \theta < 180°$

$$\sqrt{2}\sin(2\theta) = \cos(2\theta - 105°)$$

(4)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8.

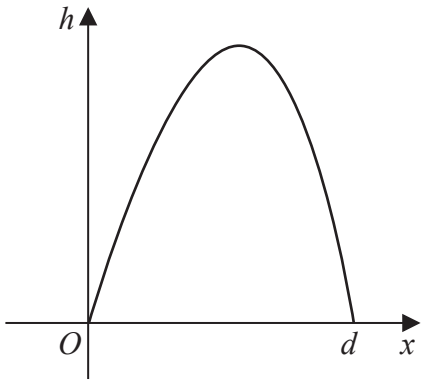

Figure 4

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, h metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball travels a horizontal distance of d metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5x\,e^{0.02x} \qquad 0 \leqslant x \leqslant d$$

Use the model to answer parts (a), (b) and (c).

(a) Find the value of d, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

(b) Show that the maximum value of h occurs when

$$x = 50\ln\left(\frac{150}{x + 50}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = 50\ln\left(\frac{150}{x_n + 50}\right) \qquad \text{with } x_1 = 30$$

(c) (i) find the value of x_2 to 2 decimal places,

(ii) find, by repeated iteration, the horizontal distance travelled by the golf ball before it reaches its maximum height. Give your answer to 2 decimal places.

(3)

Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9.

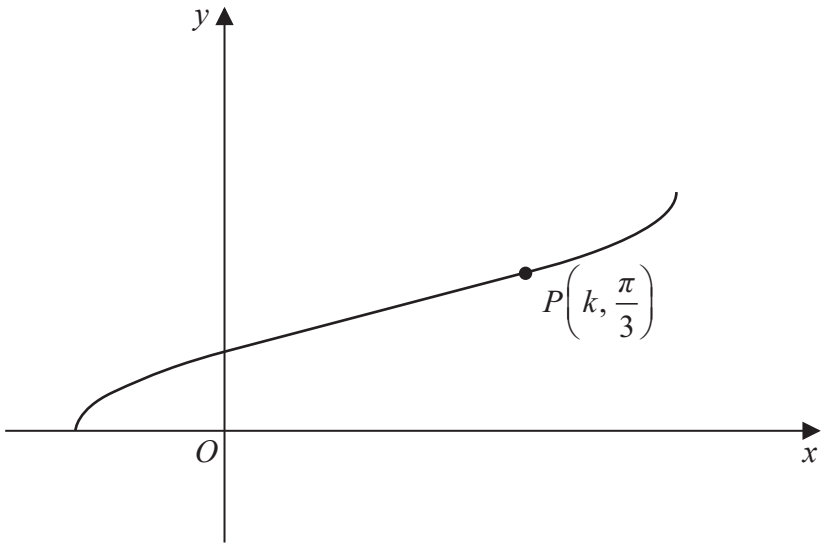

Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{dx}{dy}$ in terms of y

(ii) Hence show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x-axis at the point N.

(c) Find the exact area of triangle OPN, where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)

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Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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