British Maths

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Time 1 hour 30 minutes

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over

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Pearson

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British Maths

1. Given that k is a constant and the binomial expansion of

$$\sqrt{1+kx} \qquad |kx| < 1$$

in ascending powers of x up to the term in x^3 is

$$1 + \frac{1}{8}x + Ax^2 + Bx^3$$

(a) (i) find the value of k,

(ii) find the value of the constant A and the constant B.

(5)

(b) Use the expansion to find an approximate value to $\sqrt{1.15}$

Show your working and give your answer to 6 decimal places.

(2)

(a) (i) $(1+kx)^{1/2} = 1 + \frac{1}{2}kx + \frac{\frac{1}{2}(-\frac{1}{2})}{2!}(kx)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(kx)^3 + \ldots$

$= 1 + \frac{1}{2}kx - \frac{1}{8}k^2x^2 + \frac{1}{16}k^3x^3 +$

$\frac{1}{2}k = \frac{1}{8} \qquad k = \frac{1}{4}$

(ii) $A = -\frac{1}{8}\left(\frac{1}{4}\right)^2 = -\frac{1}{128}$

$B = \frac{1}{16}\left(\frac{1}{4}\right)^3 = \frac{1}{1024}$

(b) $1 + \frac{1}{4}x = 1.15 \qquad x = \frac{3}{5}$

$\sqrt{1.15} = 1 + \frac{1}{2}\left(\frac{1}{4}\right)\left(\frac{3}{5}\right) - \frac{1}{8}\left(\frac{1}{4}\right)^2\left(\frac{3}{5}\right)^2 + \frac{1}{16}\left(\frac{1}{4}\right)^3\left(\frac{3}{5}\right)^3$

$= 1.072398$

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2.

Diagram **NOT** drawn to scale

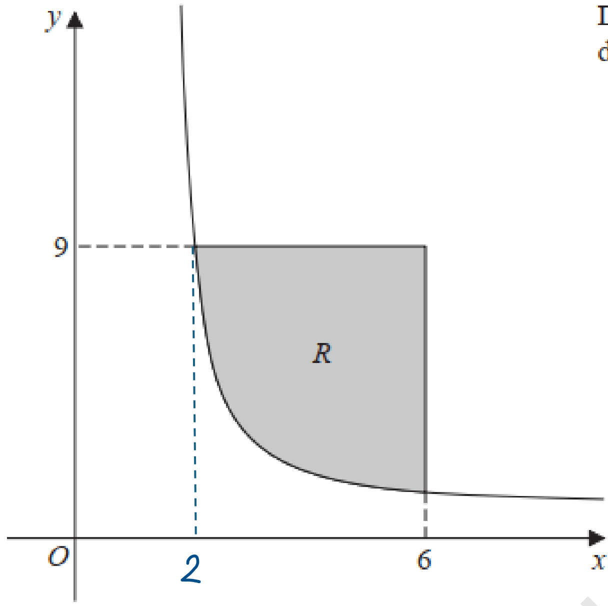


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{9}{(2x-3)^{1.25}} \qquad x > \frac{3}{2}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $y = 9$ and the line with equation $x = 6$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

Find, by algebraic integration, the exact volume of the solid generated.

(7)

$9 = \frac{9}{(2x-3)^{1.25}}$ $\qquad (2x-3)^{1.25} = 1$

$2x - 3 = 1 \qquad x = 2$

Volume of cylinder $= \pi \times 9^2 \times 4 = 324\pi$

Volume under curve $= \pi \int_2^6 y^2 \, dx$

$= \pi \int_2^6 \frac{81}{(2x-3)^{2.5}} \, dx = 81\pi \int_2^6 (2x-3)^{-2.5} \, dx$

Question 2 continued

$$= 81\pi \left[\frac{(2x-3)^{-1.5}}{-1.5 \times 2} \right]_2^6$$

$$= -27\pi \left[\frac{1}{(2x-3)^{1.5}} \right]_2^6 = -27\pi \left(\frac{1}{27} - 1 \right)$$

$$= 26\pi$$

Exact volume $= 324\pi - 26\pi = 298\pi$

(Total 7 marks)

Q2

3.

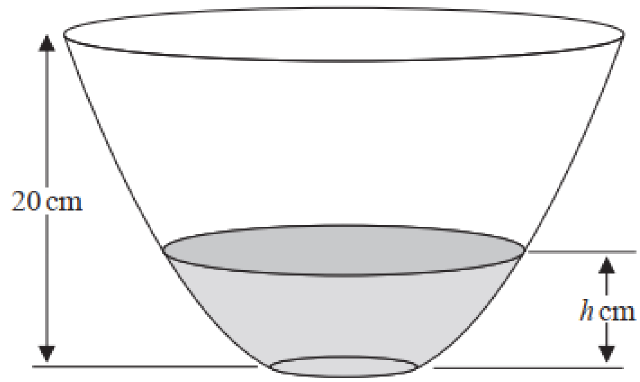


Figure 2

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is h cm, the volume of water in the bowl, V cm^3, is modelled by the equation

$$V = \frac{1}{3}h^2(h+4) \qquad 0 \leqslant h \leqslant 20$$

Given that the water flows into the bowl at a constant rate of 160 cm^3 s^{-1}, find, according to the model,

(a) the time taken to fill the bowl,

(2)

(b) the rate of change of the depth of the water, in cm s^{-1}, when $h = 5$

(5)

(a) volume of bowl $= \frac{1}{3} \times 20^2(20+4) = 3200$ cm^3

time to fill the bowl $= \frac{3200}{160} = 20$ s

(b) Rate of change of the depth $= \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{dv}{dt} \times \frac{dh}{dv}$

$\frac{dv}{dt} = 160$

Question 3 continued

$$V = \frac{1}{3}h^2(h+4) \qquad 0 \leqslant h \leqslant 20$$

$$\frac{dV}{dh} = \frac{1}{3}h^2(1) + (h+4) \times \frac{2}{3}h$$

$$= \frac{h^2}{3} + \frac{2}{3}h^2 + \frac{8}{3}h = h^2 + \frac{8}{3}h$$

$$\frac{dh}{dV} = \frac{1}{h^2 + \frac{8}{3}h}$$

$$\frac{dh}{dt} = 160 \times \frac{1}{h^2 + \frac{8}{3}h}$$

At $h = 5$ $\quad \frac{dh}{dt} = \frac{96}{23}$ cm s^{-1}

Q3

(Total 7 marks)

Leave blank

4. Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}}\,dx$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where a and b are integers to be found.

(Solutions relying entirely on calculator technology are not acceptable.)

(8)

$u = x^{\frac{1}{2}}$ $\quad du = \frac{1}{2} x^{-\frac{1}{2}} dx \quad dx = 2x^{\frac{1}{2}} du$

$= 2u\, du$

At $x=1$ $u=1$, at $x=4$ $u=2$

$$I = \int_1^2 \frac{10}{5u^2 + 2u^2 \cdot u} \times 2u\, du$$

$$= \int_1^2 \frac{20}{5u + 2u^2} du = \int_1^2 \frac{20}{u(5+2u)} du$$

$$\frac{20}{u(5+2u)} = \frac{A}{u} + \frac{B}{5+2u} = \frac{A(5+2u) + Bu}{u(5+2u)}$$

$A(5+2u) + Bu = 20$

at $u=0$ $A=4$, at $u = \frac{-5}{2}$ $\frac{-5}{2}B = 20$ $B = -8$

$$I = \int_1^2 \left(\frac{4}{u} - \frac{8}{5+2u}\right) du = 4\int_1^2 \left(\frac{1}{u} - \frac{2}{5+2u}\right) du$$

$$= 4\left[\ln u - \ln(5+2u)\right]_1^2 = 4\left[(\ln 2 - \ln 9) - (\ln 1 - \ln 7)\right] = 4\left(\ln\frac{2}{9} - \ln\frac{1}{7}\right)$$

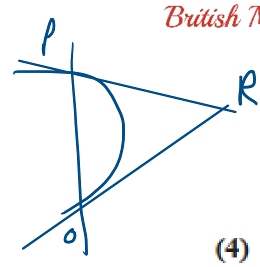
$$= 4\ln\left(\frac{2}{9} \times \frac{7}{1}\right) = 4\ln\left(\frac{14}{9}\right)$$

5. A curve has equation

$$y^2 = ye^{-2x} - 3x$$

(a) Show that

$$\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y}$$



(4)

The curve crosses the y-axis at the origin and at the point P.

The tangent to the curve at the origin and the tangent to the curve at P meet at the point R.

(b) Find the coordinates of R.

(5)

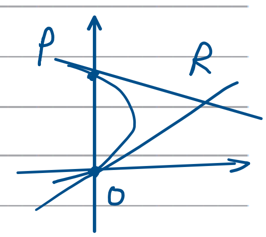
(a) $2y\frac{dy}{dx} = ye^{-2x}(-2) + e^{-2x}.\frac{dy}{dx} - 3$

$2y\frac{dy}{dx} - e^{-2x}\frac{dy}{dx} = -2ye^{-2x} - 3$

$\frac{dy}{dx}(2y - e^{-2x}) = -2ye^{-2x} - 3$

$\frac{dy}{dx} = \frac{-2ye^{-2x} - 3}{2y - e^{-2x}} = \frac{-(2ye^{-2x} + 3)}{-(e^{-2x} - 2y)}$

$= \frac{2ye^{-2x} + 3}{e^{-2x} - 2y}$



(b) At $x = 0$ $\quad y^2 = y$

$y^2 - y = 0 \quad y(y-1) = 0$

so $y = 0$ at the origin or $y = 1$ at P

$P(0,1)$

tangent at $(0,0)$ $m = \frac{0+3}{1-0} = 3$

Question 5 continued

Eqn. of OR : $y = 3x$

tangent at P $(0,1)$ $m = \dfrac{2+3}{1-2} = -5$

Eqn. of PR : $y = -5x + 1$

So for R $3x = -5x + 1$ $8x = 1$ $x = \frac{1}{8}$

$y = 3 \times \frac{1}{8} = \frac{3}{8}$

$R = \left(\frac{1}{8}, \frac{3}{8}\right)$

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Q5

(Total 9 marks)

6.

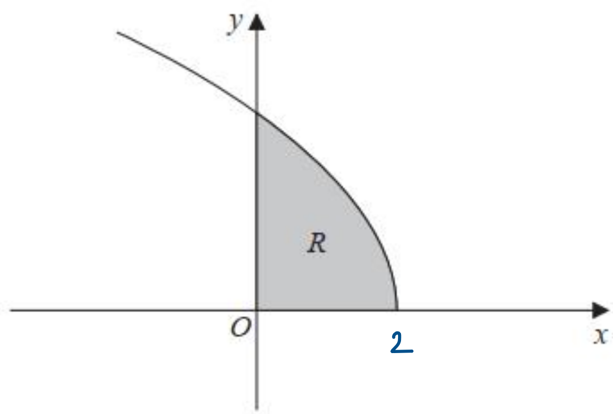


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the y-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t \cos t \, dt$

(ii) Hence find, by algebraic integration, the exact value of the area of R.

(6)

(b) Show that all points on C satisfy $y = \sqrt{ax+b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = f(x)$ where f is the function

$$f(x) = \sqrt{ax+b} \qquad -2 \leqslant x \leqslant 2$$

and a and b are the constants found in part (b).

(c) State the range of f.

(1)

(a)(i) At $y=0$ $\quad 4\sin t = 0 \quad \sin t = 0 \quad t = 0$

$x = 2\cos 0 = 2 \qquad \frac{dx}{dt} = -2\sin 2t \times 2 = -4\sin 2t$

$R = \int_0^2 y\,dx = \int 4\sin t(-4\sin 2t)\,dt$

Question 6 continued

At $x=0$ $\quad 2\cos 2t = 0 \quad \cos 2t = 0 \quad 2t = \pi/2$

$t = \pi/4$

At $x=2$ $\quad 2\cos 2t = 2 \quad \cos 2t = 1 \quad 2t = 0 \quad t = 0$

$$R = \int_{\pi/4}^{0} -16 \sin t \times \sin 2t \, dt$$

$$= \int_{0}^{\pi/4} 16 \sin t \times 2 \sin t \cos t \, dt$$

$$= \int_{0}^{\pi/4} 32 \sin^2 t \cos t \, dt$$

(ii)

	D	I
$+$	$32\sin^2 t$	$\cos t$
$-$	$64 \sin t \cos t$	$\sin t$

$$I = 32\sin^2 t (\sin t) + \int 64 \sin^2 t \cos t \, dt$$

$$= 32 \sin^3 t \quad 2\int 32 \sin^2 t \cos t \, dt$$

$$= 32 \sin^3 t \quad 2I$$

$$3I = 32 \sin^3 t$$

$$I = \left[\frac{32}{3} \sin^3 t\right]_0^{\pi/4} = \frac{32}{3} \sin^3 \frac{\pi}{4} - \frac{32}{3} \sin^3 0$$

$$R = \frac{32}{3} \left(\frac{\sqrt{2}}{2}\right)^3 = \frac{8\sqrt{2}}{3}$$

Leave blank

Question 6 continued

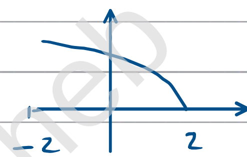
(b) $x = 2\cos 2t \qquad \cos 2t = \frac{x}{2}$

$1 - 2\sin^2 t = \frac{x}{2} \qquad \sin^2 t = \frac{1}{2} - \frac{x}{4}$

$\sin t = \sqrt{\frac{1}{2} - \frac{x}{4}} = \sqrt{\frac{2-x}{4}}$

$y = 4\sin t = 4\sqrt{\frac{2-x}{4}} = \sqrt{\frac{16}{4}(2-x)}$

$= \sqrt{8-4x} = \sqrt{-4x+8}$



(c) $-2 \le x \le 2$

At $x = -2 \qquad y = \sqrt{16} = 4$

At $x = 2 \qquad y = 0$

Range of f: $0 \le f \le 4$

Q6

(Total 10 marks)

7. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

Given that $\overrightarrow{OA}$ is a unit vector parallel to l,

(a) find $\overrightarrow{OA}$

(2)

The point X lies on l.

Given that X is the point on l that is closest to the origin,

(b) find the coordinates of X.

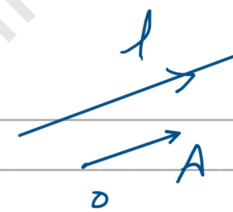
(5)

The points O, X and A form the triangle OXA.

(c) Find the exact area of triangle OXA.

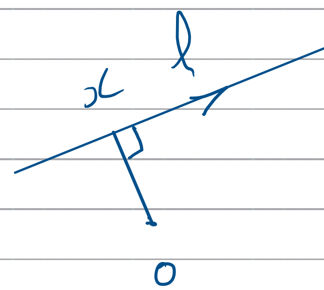
(3)

(a) $|r| = \sqrt{4^2+4^2+2^2} = 6$



$\overrightarrow{OA} = \frac{1}{6}\begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$

(b) $\overrightarrow{Ox} = \begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix}$



To get shortest dist. $\overrightarrow{Ox} \perp l$

So $\begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = 0$

Question 7 continued

Leave blank

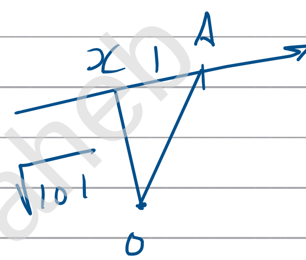
$$4(1+4\lambda)+4(-10+4\lambda)+2(-9+2\lambda)=0$$

$$4+16\lambda-40+16\lambda-18+4\lambda=0$$

$$36\lambda-54=0 \qquad \lambda=\tfrac{3}{2}$$

so $x = (7, -4, -6)$

(c) $|Ox| = \sqrt{7^2+(-4)^2+(-6)^2}$



$= \sqrt{101}$

Area of $\Delta OxA = \frac{1}{2} \times 1 \times \sqrt{101} = \frac{1}{2}\sqrt{101}$

Q7

(Total 10 marks)

8. (a) Given that $y = 1$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{6xy^{\frac{1}{3}}}{e^{2x}} \qquad y \geqslant 0$$

giving your answer in the form $y^2 = g(x)$.

(7)

(b) Hence find the equation of the horizontal asymptote to the curve with equation $y^2 = g(x)$.

(2)

(a) $\int \frac{1}{y^{1/3}}\,dy = \int \frac{6x}{e^{2x}}\,dx$

$\int y^{-1/3}\,dy = \int 6x e^{-2x}\,dx$

	D	I
+	$6x$	e^{-2x}
−	6	$-\frac{e^{-2x}}{2}$
+	0	$+\frac{e^{-2x}}{4}$

$\frac{3}{2}y^{2/3} = 6x\left(-\frac{e^{-2x}}{2}\right) - 6\left(\frac{e^{-2x}}{4}\right) + C$

$\frac{3}{2}y^{2/3} = -3xe^{-2x} - \frac{3}{2}e^{-2x} + C$

at $(0,1)$ $\quad \frac{3}{2} = -\frac{3}{2} + C \qquad C = 3$

$\frac{3}{2}y^{2/3} = -3xe^{-2x} - \frac{3}{2}e^{-2x} + 3 \qquad \times \frac{2}{3}$

$y^{2/3} = -2xe^{-2x} - e^{-2x} + 2 \qquad$ (cube)

$y^2 = \left(-2xe^{-2x} - e^{-2x} + 2\right)^3$

(b) $y^2 = \left(\frac{-2x}{e^{2x}} - \frac{1}{e^{2x}} + 2\right)^3$

As $x \to \infty \qquad \frac{-2x}{e^{2x}} \to 0 \qquad \frac{1}{e^{2x}} = 0$

so $y^2 = 2^3 \qquad y = 2^{3/2} = \sqrt{8}$

9. (i) Relative to a fixed origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.

Points A, B and C lie in a straight line, with B lying between A and C.

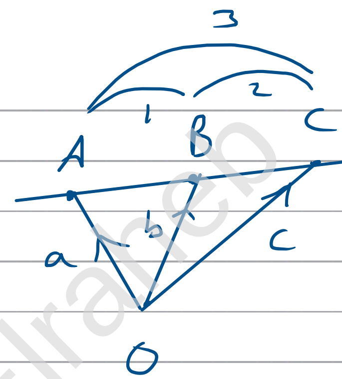
Given $AB : AC = 1 : 3$ show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$$

(3)

(ii) Given that $n \in \mathbb{N}$, prove by contradiction that if n^2 is a multiple of 3 then n is a multiple of 3

(5)



(i) $\overrightarrow{AB} = -a + b = b - a$

$\overrightarrow{BC} = 2b - 2a$

$c = b + \overrightarrow{BC} = b + 2b - 2a$

$= 3b - 2a$

(ii) If n is not a multiple of 3.

$n = 3a + 1$ or $n = 3a + 2$

$n^2 = (3a+1)^2 = 9a^2 + 6a + 1$

$= 3(3a^2 + 2a) + 1$

so not a multiple of 3

or $n^2 = (3a+2)^2 = 9a^2 + 12a + 4$

$= 3(3a^2 + 4a) + 4$

Again not a multiple of 3

Conclusion: If n is not a multiple of 3

the n^2 cannot be a multiple of 3