Pearson Edexcel International Advanced Level

Wednesday 10 January 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

P 7 3 4 8 1 A

Pearson

Write your answers in the D1 answer book for this paper.

1.

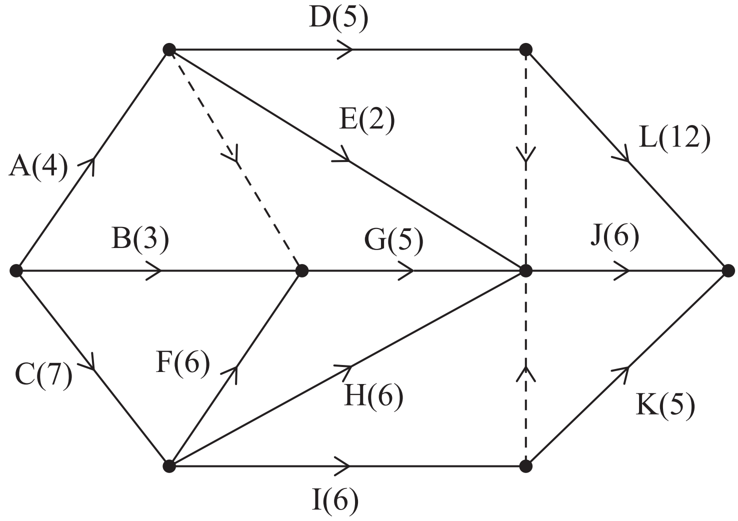

Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity. Each activity requires one worker. The project is to be completed in the shortest possible time using as few workers as possible.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(b) Calculate the total float for activity D. You must make the numbers used in your calculation clear.

(1)

(c) Calculate a lower bound for the minimum number of workers required to complete the project in the shortest possible time. You must show your working.

(2)

(d) Draw a cascade chart for this project on Grid 1 in the answer book.

(4)

(e) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities. (You do not need to provide a schedule of the activities.)

(2)

(Total for Question 1 is 13 marks)

2.

	A	B	C	D	E	F	G	H
A	–	34	29	35	28	30	37	38
B	34	–	32	28	39	40	32	39
C	29	32	–	27	33	39	34	31
D	35	28	27	–	35	38	41	36
E	28	39	33	35	–	36	33	40
F	30	40	39	38	36	–	34	39
G	37	32	34	41	33	34	–	35
H	38	39	31	36	40	39	35	–

Table 1

Table 1 represents a network that shows the travel times, in minutes, between eight towns, A, B, C, D, E, F, G and H.

(a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this network. You must clearly state the order in which you select the edges of your tree.

(3)

(b) State the weight of the minimum spanning tree.

(1)

	A	B	C	D	E	F	G	H
J	33	37	41	35	x	40	28	42

Table 2

Table 2 shows the travel times, in minutes, between town J and towns A, B, C, D, E, F, G and H.
The journey time between towns E and J is x minutes where $x > 28$

A salesperson needs to visit all of the **nine** towns, starting and finishing at J.
The salesperson wishes to minimise the total time spent travelling.

(c) Starting at J, use the nearest neighbour algorithm to find an upper bound for the duration of the salesperson's route. Write down the route that gives this upper bound.

(2)

Using the nearest neighbour algorithm, starting at E, an upper bound of 291 minutes for the salesperson's route was found.

(d) State the best upper bound that can be obtained by using this information and your answer to (c). Give the reason for your answer.

(1)

Starting by deleting J and all of its arcs, a lower bound of 264 minutes for the duration of the salesperson's route was found.

(e) Determine the value of x. You must make your method and working clear.

(3)

(Total for Question 2 is 10 marks)

3.

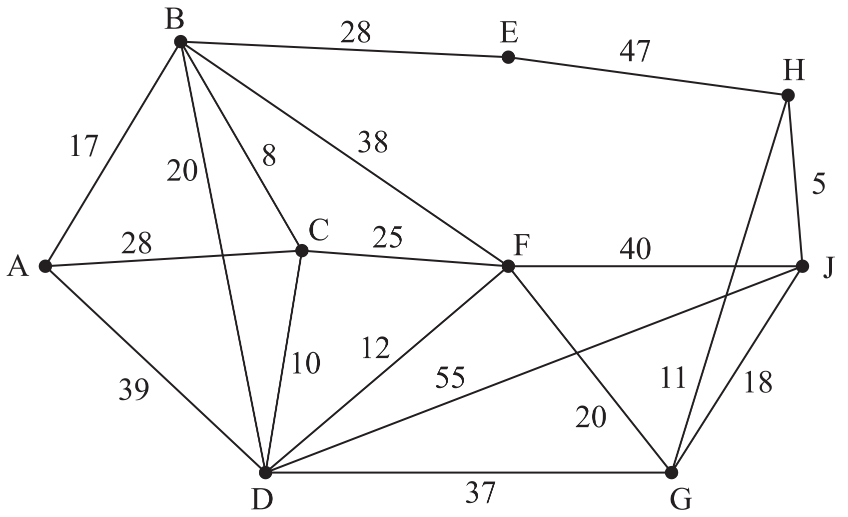

Figure 2

[*The total weight of the network is* 458]

Figure 2 represents a network of roads between nine towns, A, B, C, D, E, F, G, H and J. The number on each edge represents the length, in kilometres, of the corresponding road.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.

(ii) State the length of the shortest path from A to J.

(6)

The roads between the towns must be inspected. Claude must travel along each road at least once. Claude will start the inspection route at A and finish at J. Claude wishes to minimise the length of the inspection route.

(b) By considering the pairings of all relevant nodes, find the length of Claude's route. State the arcs that will need to be traversed twice.

(5)

If Claude does **not** start the inspection route at A and finish at J, a shorter inspection route is possible.

(c) Determine the two towns at which Claude should start and finish so that the route has minimum length. Give a reason for your answer and state the length of this route.

(3)

(Total for Question 3 is 14 marks)

4.

Activity	Immediately preceding activities
A	–
B	–
C	A, B
D	A, B
E	B
F	C, D, E
G	F
H	B
I	F
J	F
K	G
L	G, H, I, J
M	G, I

(a) Draw the activity network described in the precedence table, using activity on arc and the minimum number of dummies.

(5)

(b) Given that

- the activity network contains only one critical path
- activity E is on this critical path

state

(i) which activities could **never** be critical,

(ii) which activities **must** be critical.

(2)

(Total for Question 4 is 7 marks)

5.

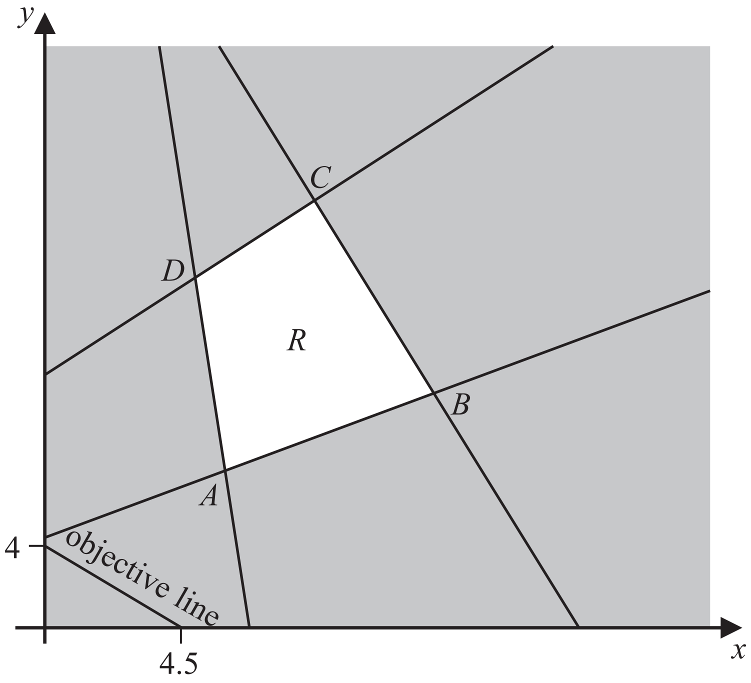

Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y.
The unshaded area, including its boundaries, forms the feasible region, R.

The four vertices of R are $A(6, 8)$, $B(13, 12)$, $C(9, 22)$ and $D(5, 18)$.

An objective line has been drawn and labelled on the graph.

When the objective function, P, is maximised, the value of P is 540

When the objective function, P, is minimised, the value of P is k

Determine the value of k. You must make your method and working clear.

(You may assume that the objective function, P, takes the form $ax + by$ where a and b are constants.)

(Total for Question 5 is 5 marks)

6. The twelve numbers in the list below are to be packed into bins of size n, where n is a positive integer.

28	31	5	25	16	35	18	22	11	27	15	13

When the first-fit bin packing algorithm is applied to the list, the following allocation is obtained.

Bin 1:	28	31	5	
Bin 2:	25	16	18	11
Bin 3:	35	22	15	
Bin 4:	27	13		

(a) Based on the packing shown above, determine the possible values of n. You must give reasons for your answer.

(3)

(b) The original list of twelve numbers is to be sorted into **ascending** order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

When the first-fit decreasing bin packing algorithm is applied to the list, the following allocation is obtained.

Bin 1:	35	31	5
Bin 2:	28	27	16
Bin 3:	25	22	18
Bin 4:	15	13	11

(c) Determine the value of n. You must give a reason for your answer.

(2)

(Total for Question 6 is 9 marks)

7. A farmer has 100 acres of land available that can be used for planting three crops: A, B and C.

It takes 2 hours to plant each acre of crop A, 1.5 hours to plant each acre of crop B and 45 minutes to plant each acre of crop C. The farmer has 138 hours available for planting.

At least one quarter of the total crops planted must be crop A.

For every three acres of crop B planted, at most five acres of crop C will be planted.

The farmer expects a profit of £160 for each acre of crop A planted, £75 for each acre of crop B planted and £125 for each acre of crop C planted.

The farmer wishes to maximise the profit from planting these three crops.

Let x, y and z represent the number of acres of land used for planting crop A, crop B, and crop C respectively.

(a) Formulate this information as a linear programming problem. State the objective, and list the constraints as simplified inequalities with integer coefficients.

(6)

The farmer decides that all 100 acres of available land will be used for planting the three crops.

(b) Explain why the maximum total profit is achieved when $-7x + 10y$ is minimised.

(2)

The farmer's decision to use all 100 acres reduces the constraints of the problem to the following:

$$x \geqslant 25$$

$$3x + 8y \geqslant 300$$

$$x + y \leqslant 100$$

$$5x + 3y \leqslant 252$$

$$y \geqslant 0$$

(c) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(d) (i) Determine the exact coordinates of each of the vertices of R.

(ii) Apply the vertex method to determine how the 100 acres should be used for planting the three crops.

(iii) Hence find the corresponding maximum expected profit.

(5)

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS